HEAVY QUARKONIUM INCLUSIVE DECAYS: THEORETICAL STATUS AND PERSPECTIVES

ANTONIO VAIRO
Theory Division, CERN, 1211 Geneva 23, Switzerland
E-mail: antonio.vairo@cern.ch

I review some recent progress and open problems in the calculation of heavy-quarkonium inclusive decay widths into light particles in the framework of QCD non-relativistic effective field theories.

1. Introduction

In recent years, the study of heavy quarkonia ($b\bar{b}$, $c\bar{c}$, ...) has gone through several theoretical and experimental advances.

From the theoretical side the introduction of non-relativistic effective field theories (EFTs) of QCD[1,2] has put our description of these systems on the solid ground of QCD. It has made it possible, in the case of several observables, to factorize the high-energy dynamics into matching coefficients calculable in perturbation theory and the non-perturbative QCD dynamics into few well-defined matrix elements to be fitted on the data or calculated on the lattice. Systematic improvements are possible, either calculating higher-order corrections in the coupling constant or adding higher-order operators.

Heavy quarkonium, being a non-relativistic bound state, is characterized by a hierarchy of energy scales m, mv and mv^2, where m is the heavy-quark mass and $v \ll 1$ the relative heavy-quark velocity. A hierarchy of EFTs may be constructed by systematically integrating out modes associated to these energy scales. Integrating out degrees of freedom of energy m, which for heavy quarks can be done perturbatively, leads to non-relativistic QCD (NRQCD)[1]. This EFT still contains the lower energy scales as dynamical degrees of freedom. In the last years, the problem of integrating out the remaining dynamical scales of NRQCD has been addressed by several groups and has now reached a solid level of conceptual understanding (an extended list of references can be found in[3]). The ultimate EFT obtained by subsequent matchings from QCD, where only the lightest degrees of freedom of energy mv^2 are left dynamical, is called potential NRQCD, pNRQCD[2]. This EFT is close to a quantum-mechanical description of the bound system and, therefore, as simple. It has been systematically explored in the dynamical regime $\Lambda_{\rm QCD} \lesssim mv^2$ in[4,5] and in the regime $mv^2 \ll \Lambda_{\rm QCD} \lesssim mv$ in[4,6,7]. The quantity $\Lambda_{\rm QCD}$ stands for the generic scale of non-perturbative physics.

From the experimental side new data have recently been produced for heavy-quarkonium observables. Measurements relevant to the determination of heavy-quarkonium inclusive decay widths have come from Fermilab (E835)[8], BES[9], CLEO[10,11] and Belle[12].

In the following I will review recent progress in our theoretical understanding of inclusive and electromagnetic heavy-quarkonium decays. I will recall the NRQCD factorization formulas in Sec. 2 and the pNRQCD factorization in Sec. 3. The presented pNRQCD formulas apply to quarkonia that fulfil $mv \gtrsim \Lambda_{\rm QCD} \gg mv^2$. In Sec. 4, I will discuss the main difficulties that still prevent us from obtaining precise predictions from the above formulas and indicate what progress has been made towards an eventual solution of these difficulties.

2. NRQCD factorization

The NRQCD factorization formulas are obtained by separating contributions coming from degrees of freedom of energy m from those coming from degrees of freedom of lower energy. In the case of heavy-quarkonium decay widths, the first are encoded in the imaginary parts of the four-fermion matching coefficients, $f, g_{1,8,ee,\gamma\gamma}(^{2S+1}L_J)$ and are ordered in powers of $\alpha_{\rm s}(m)$. The second are encoded into the matrix elements of the four-fermion operators on the heavy-quarkonium states $|H\rangle$ ($\langle\dots\rangle_H \equiv \langle H|\dots|H\rangle$). These are non-perturbative objects, which are counted, at least, in powers of mv. Matrix elements of higher dimensionality are suppressed in v. Including up to the NRQCD four-fermion operators of dimension 8, the NRQCD factorization formulas for inclusive decay widths of heavy quarkonia into light hadrons (LH) read[1,13]:

$$\Gamma(V_Q(nS) \to LH) = \frac{2}{m^2}\Bigg({\rm Im}\, f_1(^3S_1)\, \langle O_1(^3S_1)\rangle_{V_Q(nS)}$$
$$+{\rm Im}\, f_8(^3S_1)\, \langle O_8(^3S_1)\rangle_{V_Q(nS)} + {\rm Im}\, f_8(^1S_0)\, \langle O_8(^1S_0)\rangle_{V_Q(nS)}$$
$$+{\rm Im}\, g_1(^3S_1)\, \frac{\langle \mathcal{P}_1(^3S_1)\rangle_{V_Q(nS)}}{m^2} + {\rm Im}\, f_8(^3P_0)\, \frac{\langle O_8(^3P_0)\rangle_{V_Q(nS)}}{m^2}$$

$$+\mathrm{Im}\, f_8(^3P_1)\,\frac{\langle O_8(^3P_1)\rangle_{V_Q(nS)}}{m^2}+\mathrm{Im}\, f_8(^3P_2)\,\frac{\langle O_8(^3P_2)\rangle_{V_Q(nS)}}{m^2}\Bigg), \quad (1)$$

$$\Gamma(P_Q(nS)\to LH)=\frac{2}{m^2}\Bigg(\mathrm{Im}\, f_1(^1S_0)\,\langle O_1(^1S_0)\rangle_{P_Q(nS)}$$

$$+\mathrm{Im}\, f_8(^1S_0)\,\langle O_8(^1S_0)\rangle_{P_Q(nS)}+\mathrm{Im}\, f_8(^3S_1)\,\langle O_8(^3S_1)\rangle_{P_Q(nS)}$$

$$+\mathrm{Im}\, g_1(^1S_0)\,\frac{\langle \mathcal{P}_1(^1S_0)\rangle_{P_Q(nS)}}{m^2}+\mathrm{Im}\, f_8(^1P_1)\,\frac{\langle O_8(^1P_1)\rangle_{P_Q(nS)}}{m^2}\Bigg), \quad (2)$$

$$\Gamma(\chi_Q(nJS)\to LH)=\frac{2}{m^2}\Bigg(\mathrm{Im}\, f_1(^{2S+1}P_J)\,\frac{\langle O_1(^{2S+1}P_J)\rangle_{\chi_Q(nJS)}}{m^2}$$

$$+f_8(^{2S+1}S_S)\,\langle O_8(^1S_0)\rangle_{\chi_Q(nJS)}\Bigg). \quad (3)$$

At the same order the electromagnetic decay widths are given by:

$$\Gamma(V_Q(nS)\to e^+e^-)=\frac{2}{m^2}\Bigg(\mathrm{Im}\, f_{ee}(^3S_1)\,\langle O_{\mathrm{EM}}(^3S_1)\rangle_{V_Q(nS)}$$

$$+\mathrm{Im}\, g_{ee}(^3S_1)\,\frac{\langle \mathcal{P}_{\mathrm{EM}}(^3S_1)\rangle_{V_Q(nS)}}{m^2}\Bigg), \quad (4)$$

$$\Gamma(P_Q(nS)\to \gamma\gamma)=\frac{2}{m^2}\Bigg(\mathrm{Im}\, f_{\gamma\gamma}(^1S_0)\,\langle O_{\mathrm{EM}}(^1S_0)\rangle_{P_Q(nS)}$$

$$+\mathrm{Im}\, g_{\gamma\gamma}(^1S_0)\,\frac{\langle \mathcal{P}_{\mathrm{EM}}(^1S_0)\rangle_{P_Q(nS)}}{m^2}\Bigg), \quad (5)$$

$$\Gamma(\chi_Q(nJ1)\to \gamma\gamma)=2\,\mathrm{Im}\, f_{\gamma\gamma}(^3P_J)\,\frac{\langle O_{\mathrm{EM}}(^3P_J)\rangle_{\chi_Q(nJ1)}}{m^4}, \quad J=0,2\,. \quad (6)$$

The symbols V_Q and P_Q indicate respectively the vector and pseudoscalar S-wave heavy quarkonium and the symbol χ_Q the generic P-wave quarkonium (the states $\chi_Q(n10)$ and $\chi_Q(nJ1)$ are usually called $h_Q((n-1)P)$ and $\chi_{QJ}((n-1)P)$, respectively).

The operators $O, \mathcal{P}_{1,8,\mathrm{EM}}(^{2S+1}L_J)$ are the dimension 6 and 8 four-fermion operators of the NRQCD Lagrangian. They are classified in dependence of their transformation properties under colour as singlets (1) and octets (8) and under spin (S), orbital (L) and total angular momentum (J). The operators with the subscript EM are the singlet operators projected on the QCD vacuum. The explicit expressions of the operators may be found in[1,7].

The imaginary parts of the four-fermion matching coefficients have been calculated over the last twenty years to different levels of precision. In the following I will indicate, to the best of my knowledge, some recent literature where their updated value may be found. If the case arises, references to the original literature may also be found there. The imaginary parts of $f_8(^3S_1)$, $f_8(^1S_0)$, $f_8(^3P_J)$, $f_1(^{2S+1}P_J)$, and $f_1(^1S_0)$ (this originally calculated in[20]) have been calculated up to order α_s^3 in[14]. A different result for $f_1(^3P_0)$ and $f_1(^3P_2)$ is in[15]. The imaginary part of $f_1(^3S_1)$ has been calculated up to order α_s^4 in[16], the imaginary part of $g_1(^3S_1)$ at order α_s^3 may be found in[17], the imaginary part of $g_1(^1S_0)$ at order α_s^2 in[1] and the imaginary part of $f_8(^1P_1)$ at order α_s^2 in[18]. Where the electromagnetic coefficients are concerned, the imaginary part of $f_{ee}(^3S_1)$ has been calculated up to order $\alpha^2\alpha_s^2$ in[19], the imaginary parts of $f_{\gamma\gamma}(^1S_0)$ and $f_{\gamma\gamma}(^3P_{0,2})$ up to order $\alpha^2\alpha_s$ (originally calculated in[20,15]) and $g_{ee}(^3S_1)$ and $g_{\gamma\gamma}(^1S_0)$ up to order α^2 may be found in[1].

The NRQCD matrix elements are poorly known. They may be fitted on the experimental decay data, as in[21], or calculated on the lattice[22]. The matrix elements of singlet operators may be linked at leading order to the Schrödinger wave functions in the origin[1] and are often evaluated by means of potential models[23].

3. pNRQCD factorization

The pNRQCD expressions of the NRQCD matrix elements are obtained by integrating out degrees of freedom of energy larger than mv^2. Four different situations are possible: $\Lambda_{\rm QCD} \ll mv^2$, $\Lambda_{\rm QCD} \sim mv^2$, $mv \gg \Lambda_{\rm QCD} \gg mv^2$ and $\Lambda_{\rm QCD} \sim mv$. In the first situation the NRQCD matrix elements may be calculated perturbatively up to non-perturbative corrections of the form of local condensates. This situation may apply to the ground state of bottomonium. Explicit formulas have been worked out for the electromagnetic decay of the $\Upsilon(1S)$ in[24]. In the situation $\Lambda_{\rm QCD} \sim mv^2$ the NRQCD matrix elements turn out to be convolutions of non-local condensates and perturbative Green functions. In this situation, which may apply to the ground states of bottomonium and charmonium, only the real part of the spectrum has been studied[5]. Calculations of decay widths have not been done so far. In this situation, however, the state and flavour-dependent part will not factor out from the integrals. The situations $mv \gg \Lambda_{\rm QCD} \gg mv^2$

and $\Lambda_{\rm QCD} \sim mv$ have been studied in[25,7]. In principle they may apply to all heavy quarkonium states below threshold and above the ground state. Both situations give the same physical results and will be summarized in the following.[a] The main point here is that, since $\Lambda_{\rm QCD}$ is well separated from the energy scale of the bound state, mv^2, the state dependence of the NRQCD matrix elements factorizes in the quarkonium wave function, while contributions coming from excitations of order $\Lambda_{\rm QCD}$ are encoded into a few universal constants.

3.1. *pNRQCD factorization in the case* $mv \gtrsim \Lambda_{\rm QCD} \gg mv^2$

The matrix element of the NRQCD operator O on the heavy-quarkonium state $|H\rangle$ at rest, $\mathbf{P} = 0$, with quantum numbers n, j, l and s, may be written in terms of the eigenstates $|\underline{\rm k};\mathbf{x}_1\mathbf{x}_2\rangle$ of the NRQCD Hamiltonian as[7]

$$\langle O\rangle_H = \frac{1}{\langle \mathbf{P}=0|\mathbf{P}=0\rangle}\int d^3r \int d^3r' \int d^3R \int d^3R' \, \langle \mathbf{P}=0|\mathbf{R}\rangle\langle njls|\mathbf{r}\rangle \times\left[\langle \underline{0};\mathbf{x}_1\mathbf{x}_2| \int d^3\xi \; O(\boldsymbol{\xi}) \; |\underline{0};\mathbf{x}_1'\mathbf{x}_2'\rangle\right]\langle \mathbf{R}'|\mathbf{P}=0\rangle\langle \mathbf{r}'|njls\rangle \, ,$$

where $\mathbf{r} = \mathbf{x}_1 - \mathbf{x}_2$, $\mathbf{r}' = \mathbf{x}_1' - \mathbf{x}_2'$, $\mathbf{R} = (\mathbf{x}_1 + \mathbf{x}_2)/2$ and $\mathbf{R}' = (\mathbf{x}_1' + \mathbf{x}_2')/2$.

In the situation $mv \gtrsim \Lambda_{\rm QCD} \gg mv^2$ and assuming that all the higher gluonic excitations of the two heavy quarks develop a mass gap of order $\Lambda_{\rm QCD}$, the matrix element $\langle \underline{0};\mathbf{x}_1\mathbf{x}_2|\, O(\boldsymbol{\xi})\, |\underline{0};\mathbf{x}_1'\mathbf{x}_2'\rangle$ has been calculated order by order in $1/m$ in[25,7]. Once normalized to m, we get up to $\mathcal{O}(v^3 \times (\Lambda_{\rm QCD}^2/m^2, E/m))$ for the NRQCD S-wave matrix elements and up to $\mathcal{O}(v^5)$ for the NRQCD P-wave matrix elements[7]:

$$\langle O_1(^3S_1)\rangle_{V_Q(nS)} = C_A \frac{|R^V_{n0}(0)|^2}{2\pi}\left(1 - \frac{E^{(0)}_{n0}}{m}\frac{2\mathcal{E}_3}{9} + \frac{2\mathcal{E}^{(2,t)}_3}{3m^2} + \frac{c_F^2\mathcal{B}_1}{3m^2}\right), \quad (7)$$

$$\langle O_1(^1S_0)\rangle_{P_Q(nS)} = C_A \frac{|R^P_{n0}(0)|^2}{2\pi}\left(1 - \frac{E^{(0)}_{n0}}{m}\frac{2\mathcal{E}_3}{9} + \frac{2\mathcal{E}^{(2,t)}_3}{3m^2} + \frac{c_F^2\mathcal{B}_1}{m^2}\right), \quad (8)$$

$$\langle O_{\rm EM}(^3S_1)\rangle_{V_Q(nS)} = C_A \frac{|R^V_{n0}(0)|^2}{2\pi}\left(1 - \frac{E^{(0)}_{n0}}{m}\frac{2\mathcal{E}_3}{9} + \frac{2\mathcal{E}^{(2,{\rm EM})}_3}{3m^2} + \frac{c_F^2\mathcal{B}_1}{3m^2}\right), \quad (9)$$

$$\langle O_{\rm EM}(^1S_0)\rangle_{P_Q(nS)} = C_A \frac{|R^P_{n0}(0)|^2}{2\pi}\left(1 - \frac{E^{(0)}_{n0}}{m}\frac{2\mathcal{E}_3}{9} + \frac{2\mathcal{E}^{(2,{\rm EM})}_3}{3m^2} + \frac{c_F^2\mathcal{B}_1}{m^2}\right),$$

[a] Possible threshold effects have been neglected. As suggested in[26] they may be large for the $\psi(2S)$.

$$(10)$$

$$\langle O_1({}^{2S+1}P_J)\rangle_{\chi_Q(nJS)} = \langle O_{\rm EM}({}^{2S+1}P_J)\rangle_{\chi_Q(nJS)} = \frac{3}{2}\frac{C_A}{\pi}|R^{(0)\,\prime}_{n1}(0)|^2, \quad (11)$$

$$\langle \mathcal{P}_1({}^3S_1)\rangle_{V_Q(nS)} = \langle \mathcal{P}_1({}^1S_0)\rangle_{P_Q(nS)} = \langle \mathcal{P}_{\rm EM}({}^3S_1)\rangle_{V_Q(nS)}$$
$$= \langle \mathcal{P}_{\rm EM}({}^1S_0)\rangle_{P_Q(nS)} = C_A\frac{|R^{(0)}_{n0}(0)|^2}{2\pi}\left(mE^{(0)}_{n0} - \mathcal{E}_1\right), \quad (12)$$

$$\langle O_8({}^3S_1)\rangle_{V_Q(nS)} = \langle O_8({}^1S_0)\rangle_{P_Q(nS)}$$
$$= C_A\frac{|R^{(0)}_{n0}(0)|^2}{2\pi}\left(-\frac{2(C_A/2 - C_f)\mathcal{E}^{(2)}_3}{3m^2}\right), \quad (13)$$

$$\langle O_8({}^1S_0)\rangle_{V_Q(nS)} = \frac{\langle O_8({}^3S_1)\rangle_{P_Q(nS)}}{3}$$
$$= C_A\frac{|R^{(0)}_{n0}(0)|^2}{2\pi}\left(-\frac{(C_A/2 - C_f)c_F^2\mathcal{B}_1}{3m^2}\right), \quad (14)$$

$$\langle O_8({}^3P_J)\rangle_{V_Q(nS)} = \frac{\langle O_8({}^1P_1)\rangle_{P_Q(nS)}}{3}$$
$$= (2J+1)\,C_A\frac{|R^{(0)}_{n0}(0)|^2}{2\pi}\left(-\frac{(C_A/2 - C_f)\mathcal{E}_1}{9}\right), \quad (15)$$

$$\langle O_8({}^1S_0)\rangle_{\chi_Q(nJS)} = \frac{T_F}{3}\frac{|R^{(0)\,\prime}_{n1}(0)|^2}{\pi m^2}\mathcal{E}_3, \quad (16)$$

where $R^V_{n0} = R^{(0)}_{n0}(1+\mathcal{O}(v))$ is the radial part of the vector S-wave function, $R^P_{n0} = R^{(0)}_{n0}(1+\mathcal{O}(v))$ the radial part of the pseudoscalar S-wave function, $R^{(0)\,\prime}_{n1}$ the derivative of the LO P-wave function and $E^{(0)}_{n0} \simeq M_H - 2m \sim mv^2$ the LO binding energy. A consistent way to calculate the wave functions is from the real part of the pNRQCD Hamiltonian obtained up to order $1/m^2$ in[6]. Any other S-wave dimension-6 matrix element is 0 at NNLO and any other S-wave dimension-8 matrix element is 0 at LO. The symbol c_F indicates the NRQCD chromomagnetic matching coefficient, which is known at NLO from[27]. All the non-perturbative dynamics is encoded into the wave functions and the six chromoelectric and chromomagnetic correlators $\mathcal{E}_1$, $\mathcal{E}_3$, $\mathcal{E}^{(2)}_3$, $\mathcal{E}^{(2,t)}_3$, $\mathcal{E}^{(2,\rm EM)}_3$ and $\mathcal{B}_1$. The exact definition of the correlators may be found in[7]. It is important to note that, owing to the factorization of the state and flavour dependence into the wave function, on the whole set of heavy-quarkonium states below threshold the number of non-perturbative parameters needed to describe inclusive decays has diminished with respect to NRQCD, so that definite new predictions are possible. In practice, one may consider ratios where the wave-function dependence drops out[25,7] and fix the correlators either through lattice calculations or through specific models of the QCD vacuum[28] or on some set of experimental data. In particular, new predictions for P-wave bottomonium inclusive decay-width ratios were made in this way in[25,29] before the CLEO-III data were made available[11].

4. How to use the above formulas

Here I would like to point out that there are two potential problems that may still prevent us from getting precise determinations of heavy-quarkonium decay widths from the above formulas. These problems seem to be often underestimated in the existing literature. Eventually, they may lead to an underestimate of the errors associated to the calculated values of the decay widths or to the extracted values of the matrix elements and $\alpha_s(m)$.

i) It has been discussed, in particular in[30] (but see also[31]), that higher-order operators, not considered in the above formulas, can be numerically quite relevant. This may be the case particularly for charmonium, where $v_c^2 \sim 0.3$, so that relativistic corrections are large, and for P-wave decays where the above formulas provide, indeed, only the leading-order contribution in the velocity expansion. In order to be specific, in the case of the χ_c decay width into two photons the corrections of relative $\mathcal{O}(\Lambda_{\rm QCD}^2/m^2, E/m)$ to the formulas (6) are given by[30]:

$$\frac{\delta\Gamma(\chi_{c0}\to\gamma\gamma)}{\Gamma^{(0)}(\chi_{c0}\to\gamma\gamma)} \simeq -2.3\frac{E_{\chi_c}}{m_c} - 3\,\mathcal{A}\left(\frac{\Lambda_{\rm QCD}^2}{m_c^2}\right),$$

$$\frac{\delta\Gamma(\chi_{c2}\to\gamma\gamma)}{\Gamma^{(0)}(\chi_{c2}\to\gamma\gamma)} \simeq -1.0\frac{E_{\chi_c}}{m_c} - 4\,\mathcal{B}\left(\frac{\Lambda_{\rm QCD}^2}{m_c^2}\right),$$

where $\mathcal{A}\left(\Lambda_{\rm QCD}^2/m_c^2\right)$ and $\mathcal{B}\left(\Lambda_{\rm QCD}^2/m_c^2\right)$ are some specific combinations of matrix elements of order $\Lambda_{\rm QCD}^2/m_c^2$. Choosing $E_{\chi_c}/m_c \simeq 0.3$ and $|\mathcal{A},\mathcal{B}(\Lambda_{\rm QCD}^2/m_c^2)| \simeq 0.1$ (or 0.3)[b] the relative corrections may be $-(0.7\pm 0.3)$ (or $-(0.7\pm0.9)$) for $\Gamma(\chi_{c0}\to\gamma\gamma)$ and $-(0.3\pm0.4)$ (or $-(0.3\pm1.2)$) for $\Gamma(\chi_{c2}\to\gamma\gamma)$ and, therefore, potentially as large as the leading one. The two $\pm$ values refer to the two possible choices of sign for $\mathcal{A}$ and $\mathcal{B}$. Note that for some specific choice of sign, large cancellations may also occur.

The introduction of higher-order matrix elements may spoil the predictive power of the NRQCD factorization. A way out is provided by pN-

---

[b] The first choice corresponds to a hierarchy of the type $\Lambda_{\rm QCD} \sim mv^2$, the second of the type $\Lambda_{\rm QCD} \sim mv$.

RQCD, which can be used to reduce also these new matrix elements to a few universal correlators[7].

ii) The convergence of the perturbative series of the four-fermion matching coefficients is often bad. Let us consider, for instance, the following matching coefficients ($n_f = 3$):

$$\begin{aligned}
\mathrm{Im} f_1(^1S_0) &= (\ldots) \times \left(1 + 11.1\,\frac{\alpha_\mathrm{s}}{\pi}\right), \\
\mathrm{Im} f_8(^1S_0) &= (\ldots) \times \left(1 + 13.7\,\frac{\alpha_\mathrm{s}}{\pi}\right), \\
\mathrm{Im} f_8(^3S_1) &= (\ldots) \times \left(1 + 10.3\,\frac{\alpha_\mathrm{s}}{\pi}\right), \\
\mathrm{Im} f_1(^3P_0) &= (\ldots) \times \left(1 + \left(13.6 - 0.44\,\log\frac{\mu}{2m}\right)\,\frac{\alpha_\mathrm{s}}{\pi}\right), \\
\mathrm{Im} f_1(^3P_2) &= (\ldots) \times \left(1 - \left(0.73 + 1.67\,\log\frac{\mu}{2m}\right)\,\frac{\alpha_\mathrm{s}}{\pi}\right).
\end{aligned}$$

Apart from the case of $\mathrm{Im} f_1(^3P_2)$, the series in α_s of the other coefficients does not seem to converge. This behaviour may not be adjusted by a suitable choice of the factorization scale μ, which enters only in $\mathrm{Im} f_1(^3P_0)$; in this case lowering the factorization scale below $2m$ would make the convergence worse.[c] A solution may be provided by the resummation of the large contributions in the perturbative series coming from bubble-chain diagrams. This analysis has been successfully carried out in some specific cases in[33]. A general treatment is still missing, in particular in the case of P-wave decays.

Finally, I would like to note that neither contributions coming from higher-order matrix elements of the type described in (*i*) nor resummations of large contributions in the perturbative series of the matching coefficients discussed in (*ii*) have been considered in recent determinations of α_s (e.g. in[32,21]) and of the NRQCD matrix elements (e.g. in[21]) from the charmonium P-wave decay data.

5. Conclusions

The progress in our understanding of non-relativistic effective field theories makes it possible to move beyond *ad hoc* phenomenological models and have a unified description of the different heavy-quarkonium observables, so that the same quantities determined from a set of data may be used in order to

---

[c] In the case of the coefficient $\mathrm{Im} f_{ee}(^3S_1)$, which presents a similar problem, it has been noted by Beneke et al. in[19] that in the bottomonium case a suitable choice of the factorization scale may adjust the convergence of the series up to order $\alpha^2\alpha_\mathrm{s}^2$.

describe other sets. Moreover, predictions based on non-relativistic EFTs are conceptually solid, and systematically improvable. Therefore, the recent progress in the measurement of several heavy-quarkonium observables makes it meaningful to ask whether their precise theoretical determination is feasible.

Here, I have focused on inclusive and electromagnetic heavy-quarkonium decays. In the framework of NRQCD, heavy-quarkonium decay widths may be expressed in terms of matrix elements, which, in the case of matrix elements of singlet operators, may be linked to the wave function in the origin. This is also the case of some matrix elements entering in the description of production processes[1]. In the framework of pNRQCD also some octet matrix elements may be expressed in terms of the wave function in the origin and some universal non-local gluon-field correlators. These correlators enter also in the expression of the masses of some heavy-quarkonium states[5,34]. Specific predictions have been discussed in[25,7]. There are, however, still some difficulties that hinder in several cases precise predictions of heavy-quarkonium decay observables. These are essentially related to the control of higher-order corrections in the velocity and α_s expansion and have been addressed in the last section. In principle, the tools to overcome these difficulties already exist, so that progress in the field is expected from the coordinated effort of the heavy-quarkonium community in the near future[35].

Acknowledgements

I acknowledge the support of the European Community through a Marie-Curie fellowship, contract No. HPMF-CT-2000-00733.

References

1. G.T. Bodwin, E. Braaten and G.P. Lepage, Phys. Rev. **D51**, 1125 (1995); Erratum, *ibid.* **D55**, 5853 (1997).
2. A. Pineda and J. Soto, Nucl. Phys. Proc. Suppl. **64**, 428 (1998).
3. A. Pineda, Nucl. Phys. Proc. Suppl. **93**, 188 (2001); A. H. Hoang, hep-ph/0204299.
4. N. Brambilla, A. Pineda, J. Soto and A. Vairo, Nucl. Phys. **B566**, 275 (2000).
5. N. Brambilla, A. Pineda, J. Soto and A. Vairo, Phys. Lett. **B470**, 215 (1999).
6. N. Brambilla, A. Pineda, J. Soto and A. Vairo, Phys. Rev. **D63**, 014023 (2001); A. Pineda and A. Vairo, Phys. Rev. **D63**, 054007 (2001); Erratum, *ibid.* **D64**, 039902 (2001).
7. N. Brambilla, D. Eiras, A. Pineda, J. Soto and A. Vairo, hep-ph/0208019, to appear in Phys. Rev. **D**.

8. S. Bagnasco *et al.*, Phys. Lett. **B533**, 237 (2002).
9. J.Z. Bai *et al.*, Phys. Lett. **B550**, 24 (2002).
10. B.I. Eisenstein *et al.*, Phys. Rev. Lett. **87**, 061801 (2001).
11. D. Cinabro *et al.*, hep-ex/0207062.
12. K. Abe *et al.*, Phys. Lett. **B540**, 33 (2002).
13. G.T. Bodwin, E. Braaten and G.P. Lepage, Phys. Rev. **D46**, R1914 (1992).
14. A. Petrelli, M. Cacciari, M. Greco, F. Maltoni and M.L. Mangano, Nucl. Phys. **B514**, 245 (1998).
15. R. Barbieri, M. Caffo, R. Gatto and E. Remiddi, Phys. Lett. **B95**, 93 (1980); Nucl. Phys. **B192**, 61 (1981).
16. P.B. Mackenzie and G.P. Lepage, Phys. Rev. Lett. **47**, 1244 (1981).
17. M. Gremm and A. Kapustin, Phys. Lett. **B407**, 323 (1997).
18. F. Maltoni, PhD thesis (Univ. of Pisa, 1999) [http://web.hep.uiuc.edu/home/maltoni/thesis.ps].
19. A. Czarnecki and K. Melnikov, Phys. Rev. Lett. **80**, 2531 (1998); M. Beneke, A. Signer and V.A. Smirnov, Phys. Rev. Lett. **80**, 2535 (1998).
20. R. Barbieri, G. Curci, E. d'Emilio and E. Remiddi, Nucl. Phys. **B154**, 535 (1979); K. Hagiwara, C.B. Kim and T. Yoshino, Nucl. Phys. **B177**, 461 (1981).
21. F. Maltoni, hep-ph/0007003.
22. G.T. Bodwin, D.K. Sinclair and S. Kim, Int. J. Mod. Phys. **A12**, 4019 (1997); Phys. Rev. **D65**, 054504 (2002).
23. E.J. Eichten and C. Quigg, Phys. Rev. **D52**, 1726 (1995).
24. S. Titard and F.J. Yndurain, Phys. Rev. **D49**, 6007 (1994).
25. N. Brambilla, D. Eiras, A. Pineda, J. Soto and A. Vairo, Phys. Rev. Lett. **88**, 012003 (2002).
26. Y.Q. Chen and E. Braaten, Phys. Rev. Lett. **80**, 5060 (1998).
27. G. Amoros, M. Beneke and M. Neubert, Phys. Lett. **B401**, 81 (1997).
28. A. Di Giacomo, H.G. Dosch, V.I. Shevchenko and Yu.A. Simonov, Phys. Rep. **372**, 319 (2002); M. Baker, N. Brambilla, H. G. Dosch and A. Vairo, Phys. Rev. **D58**, 034010 (1998); G. Bali, N. Brambilla and A. Vairo, Phys. Lett. **B421**, 265 (1998); M. D'Elia, A. Di Giacomo and E. Meggiolaro, Phys. Lett. **B408**, 315 (1997).
29. A. Vairo, hep-ph/0205128.
30. J.P. Ma and Q. Wang, Phys. Lett. **B537**, 233 (2002).
31. G.T. Bodwin and A. Petrelli, Phys. Rev. **D66**, 094011 (2002).
32. M. Ambrogiani *et al.*, Phys. Rev. **D62**, 052002 (2000).
33. G.T. Bodwin and Y.Q. Chen, Phys. Rev. **D64**, 114008 (2001); Phys. Rev. **D60**, 054008 (1999); E. Braaten and Y.Q. Chen, Phys. Rev. **D57**, 4236 (1998); G.T. Bodwin at this conference.
34. N. Brambilla and A. Vairo, hep-ph/0004192.
35. http://alephwww.physik.uni-siegen.de/∼quarkonium/.